Model Answer (suggestive)

AU - 6954

B.Sc. (First semester) Examination, 2014

Basic mathematics (Forestry)

1. (i) $(i)^{147} = i(i)^{146} = i(i^2)^{73}$

$= i(-1)^{73} = -i$

(ii) $i^5 + i^6 + i^7 = i(i^2)^2 + (i^2)^3 + i(i^2)^3$

$= i(-1)^2 + (-1)^3 + i(-1)^3$

$= \cancel{i} - 1 - \cancel{i}$

$= -1$

(iii) $\bar{z} = x - iy$

(iv) $3i^3(15i^6) = 45i^9$

$= 45\, i(i^2)^4$

$= 45i$

(v) cube roots of unity are $1, \omega, \omega^2$ and $1+\omega+\omega^2 = 0, \omega^3 = 1$

(vi) $A.M. = \frac{a+b}{2}$

(vii) $l = a + (n-1)d$

(viii) $a_n = n(n+2)$

$a_1 = 3,\ a_2 = 8,\ a_3 = 15$

~~(viii)~~

(ix). $\frac{a}{r}, a, ar$

(x). $1 + \tan^2\theta = \sec^2\theta$.

2 (a). Let $z = 4 - 3i$

Then $z^{-1} = \frac{1}{z} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$

$= \frac{4+3i}{16 - 9i^2} = \frac{4}{25} + i\frac{3}{25}$

P.T.O.

2(b). Let $z = -1 + i\sqrt{3}$

Then $x = -1$, $y = \sqrt{3}$

Modulus of $z = |z| = \sqrt{x^2+y^2}$

$= \sqrt{(-1)^2+(\sqrt{3})^2} = 2$

argument $(\theta) = \tan^{-1}(y/x)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = \tan^{-1}(-\sqrt{3})$

$= \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$= \frac{2\pi}{3}$ or $-\pi/3$

3(a). Let three no. in A.P. be $a-d, a, a+d$

Then $a-d+a+a+d = 27 \Rightarrow 3a = 27$

$\Rightarrow a = 9$

and $(a-d)a(a+d) = 504$

$\Rightarrow a(a^2-d^2) = 504$

$9(9^2-d^2) = 504$

$d^2 = 81 - 56 = 25$

$\Rightarrow d = \pm 5$

no. 4, 9, 14 or 14, 9, 4

(b). Let three no. in G.P. be $\frac{a}{r}, a, ar$

then $\frac{a}{r} + a + ar = 19 \longrightarrow (1)$

and $\frac{a}{r} \cdot a \cdot ar = 216 \Rightarrow a^3 = 216$

$\Rightarrow a = 6$

P.T.O.

Put $a=6$ in eqn ①

$$\frac{6}{r}+6+6r=19 \Rightarrow 6+6r+6r^2=19r$$

$$\Rightarrow 6r^2-13r+6=0 \Rightarrow 6r^2-9r-4r+6=0$$

$$\Rightarrow 3r(2r-3)-2(2r-3)=0$$

$$(2r-3)(3r-2)=0 \Rightarrow r=3/2 \text{ or } 2/3$$

when $a=6$, $r=3/2$ then no. $=4, 6, 9$

when $a=6$, $r=2/3$ then no. $=9, 6, 4$.

4. Given $\left(\frac{a}{x}+\frac{x}{a}\right)^{10}$

Then $a=\frac{a}{x}$, $b=\frac{x}{a}$, $n=10$ (even)

$\therefore$ middle term $=\left(\frac{n}{2}+1\right)^{th} = \left(\frac{10}{2}+1\right)^{th}$

$= 6^{th}$ term

$\because (r+1)^{th} = {}^nC_r \cdot a^{n-r} \cdot b^r$

$$6^{th} = (5+1)^{th} = {}^{10}C_5 \cdot \left(\frac{a}{x}\right)^{10-5} \cdot \left(\frac{x}{a}\right)^5$$

$$= \frac{10!}{5!\,5!} = \frac{\cancel{10}^2 \times 9 \times \cancel{8}^2 \times 7 \times \cancel{6}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= \underline{\underline{252}}$$

5. L.H.S. $= \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{\sin^3 A - \cos^3 A}{\cos A \sin A (\sin A - \cos A)}$$

$$= \frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \cos^2 A + \sin A \cdot \cos A)}{\cos A \sin A \cancel{(\sin A - \cos A)}}$$

P.T.O.

$$= \frac{1 + \sin A \cos A}{\sin A \cos A} = \sec A \operatorname{cosec} A + 1.$$

6 (a)

(i) $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

(ii) $\cos 75^\circ = \cos(45^\circ + 30^\circ)$

$$= \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{3}}$$

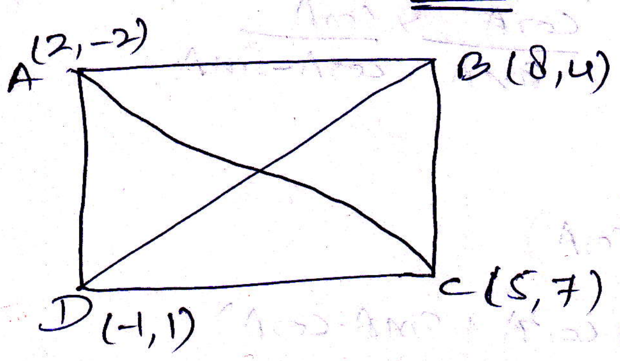
(b). Given points $(-3, -2)$ and $(-6, 7)$

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-6+3)^2 + (7+2)^2}$$

$$= \sqrt{9 + 81} = \sqrt{90}$$

$$= 3\sqrt{10}$$

7. 

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

P.T.O.

$AB = \sqrt{(8-2)^2+(4+2)^2} = \sqrt{36+36} = \sqrt{72}$

$BC = \sqrt{(5-8)^2+(7-4)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(-1-5)^2+(7-1)^2} = \sqrt{36+36} = \sqrt{72}$

$DA = \sqrt{(2+1)^2+(-2-1)^2} = \sqrt{9+9} = \sqrt{18}$

$AC = \sqrt{(5-2)^2+(7+2)^2} = \sqrt{9+81} = \sqrt{90}$

$BD = \sqrt{(8+1)^2+(4-1)^2} = \sqrt{81+9} = \sqrt{90}$

$\because AB = CD,\ BC = DA,\ AC = BD$

$\therefore$ above points are the angular points of a rectangle.

8. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

Now,
$$|A| = 1(-9+8) - 1(6-12) - 1(-4+9)$$
$$= -1 + 6 - 5$$
$$= 0$$

$\because |A| = 0$

$\therefore$ inverse does not exists.

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Dsingh.